Graphing Cheat Sheet

- First Derivative Information:
  - Critical points: points in the domain of *f(x)* where the derivative is zero or does not exist
  - If $f'(x) < 0$ then f is decreasing and if $f'(x) > 0$ then f is increasing.
  - Maximum/minimum Point
    - If $f'(x)$ changes sign from positive to negative (negative to positive) at $x = p$ (in the domain of $f(x)$) then that point is a local maximum (minimum) point. Note that when $f'(p) = 0$ it does not necessarily mean that $x = p$ is a local max/min point.
    - For absolute maximum/minimum in the interval [a,b] you need to compare the y-values of local max/min with those of the boundary values; i.e., $f(a)$ and $f(b)$. The highest (lowest) will be the absolute max (min).
    - Boundary points cannot be local max/min.

- Second Derivative Information:
  - If $f''(x) > 0$ then f is concave up and if $f''(x) < 0$ then f is concave down
  - If $f''(x)$ changes sign at point x=p (in the domain of $f(x)$) then that point is an inflection point. Note that when $f''(p) = 0$ it does not necessarily mean that $x = p$ is an inflection point.
  - Second derivative can be used as a test for finding max/min If you had more than one critical point. Given that $f'(p) = 0$ or $f'(p) = DNE$ then
    - Second derivative test [if it exists]: If $f''(p) < 0$, it's a max (concave down). If $f''(p) > 0$, it's a min (concave up).

© Calculus Captain. All Rights Reserved.

Ø How to Build a Sign Chart

1. List the important points: zeros of the derivative, and points where it's undefined.
2. Order them on a number line. Be mindful of the domain.
3. Test one value in each interval, or use the sign of each factor.
4. Mark + or – in each interval, then translate:
    - for f′: ↗ or ↘
    - for f″: ∪ or ∩

Tip: factor completely first. Factors that are always positive (for example e^x or $x^2 + 1$) never change the sign, so you can ignore them.

Asymptote Shortcuts (Rational Functions)
For degree of numerator n and degree of denominator m:

- n < m: horizontal asymptote y = 0
- n = m: horizontal asymptote y = (ratio of leading coefficients)
- n > m: no horizontal asymptote

Ø Special Functions

- Exponential, e^x: always positive, with a horizontal asymptote y = 0 as $x \to -\infty$.
- Logarithmic, $\ln x$:
    - the domain is x > 0, with a vertical asymptote at x = 0
    - for ln(g(x)), you need g(x) > 0
- Roots, e.g. $\sqrt[3]{x^2}$: f′ can be undefined, which gives a vertical tangent.

Ø Common Mistakes That Cost Marks:

- ❌ Forgetting the domain: listing a critical number or inflection point where f isn't defined.
- ❌ Calling every f′ = 0 point a max/min without checking for a sign change.
- ❌ Calling every f″ = 0 point an inflection point without checking for a sign change.
- ❌ Missing important points where f′ or f″ is DNE.
- ❌ Checking only one end behavior: horizontal asymptotes can differ as $x \to +\infty$ and $x \to -\infty$ especially in exponential functions.
- ❌ Algebra slips in f' or f″: simplify f′ fully before you differentiate again.
- ❌ A sketch that contradicts your own sign charts, for example a curve going up where f′ < 0.

© Calculus Captain. All Rights Reserved.