Graphing Starter Pack Solutions

1. $f(x) = \frac{x-2}{x^2-x+1} \rightarrow f'(x) = \frac{1(x^2-x+1)-(x-2)(2x-1)}{(x^2-x+1)^2} = \frac{(x^2-x+1)-(2x^2-5x+2)}{(x^2-x+1)^2} = \frac{-x^2+4x-1}{(x^2-x+1)^2}$

Now:

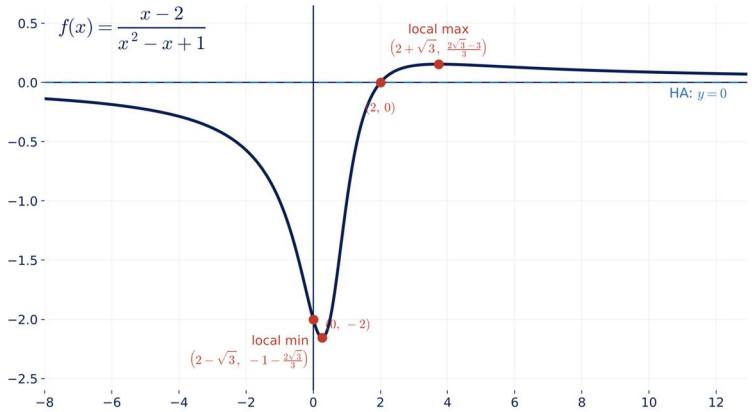
$$f'(x) = 0 \rightarrow -x^2 + 4x - 1 = 0 \rightarrow x = \frac{-4 \pm \sqrt{16-4}}{-2} = \frac{-4 \pm 2\sqrt{3}}{-2} = 2 \pm \sqrt{3}$$
$$f'(x) = DNE \rightarrow x^2 - x + 1 = 0 \rightarrow Not\ Possible$$

x	$-\infty$	$2-\sqrt{3}$	$2+\sqrt{3}$	$+\infty$
f'	$-$	$+$	$-$	
f	↘	↗	↘	
	Local min		Local max	

Interval of decreasing: $(-\infty, 2-\sqrt{3}] \cup [2+\sqrt{3}, +\infty)$
Interval of increasing: $[2-\sqrt{3}, 2+\sqrt{3}]$

Local min: $f(2-\sqrt{3}) = \frac{(2-\sqrt{3})-2}{(2-\sqrt{3})^2-(2-\sqrt{3})+1} = \frac{-\sqrt{3}}{(7-4\sqrt{3})-(2-\sqrt{3})+1} = \frac{-\sqrt{3}}{6-3\sqrt{3}} \rightarrow \left(2-\sqrt{3}, \frac{-\sqrt{3}}{6-3\sqrt{3}}\right)$
Local max: $f(2+\sqrt{3}) = \frac{(2+\sqrt{3})-2}{(2+\sqrt{3})^2-(2+\sqrt{3})+1} = \frac{\sqrt{3}}{(7+4\sqrt{3})-(2+\sqrt{3})+1} = \frac{\sqrt{3}}{6+3\sqrt{3}} \rightarrow \left(2+\sqrt{3}, \frac{\sqrt{3}}{6+3\sqrt{3}}\right)$

And here is the graph for the reference (the y-value of max and min has been rationalized):



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2. For vertical asymptotes of $\frac{4x^2+3x+1}{x^2-4}$ we set the denominator equal to zero:

$$x^2 - 4 = 0 \rightarrow x = 2 \; and \; x = -2$$

We just have to ensure none of the aforementioned x values do not make the numerator also approach zero. If they do, we have to compute the limit of the function and ensure it approaches infinity. If $f(x)$ does not approach infinity when x approaches that value, then it is not a vertical asymptote. In this case neither $x = 2$ nor $x = -2$ do not make numerator equal to zero. So, both of them are vertical asymptotes.

For horizontal asymptotes:

$$\lim_{x\to\infty} \frac{4x^2 + 3x + 1}{x^2 - 4} = \lim_{x\to\infty} \frac{x^2(4 + \frac{3}{x} + \frac{1}{x^2})}{x^2(1 - \frac{4}{x^2})} = \lim_{x\to\infty} \frac{x^2(4 + 0 + 0)}{x^2(1 - 0)} = 4$$

So, $y = 4$ is a horizontal asymptote. Note that because we have a rational function, the limit of that as $x \rightarrow -\infty$ will also be the same.

3. $f(x) = \frac{2x+2}{x-3} \rightarrow f'(x) = \frac{2(x-3)-(2x+2)(1)}{(x-3)^2} = \frac{2x-6-2x-2}{(x-3)^2} = \frac{-8}{(x-3)^2}$

Now:

$$f'(x) = 0 \rightarrow \; Not \; Possible$$
$$f'(x) = DNE \rightarrow x - 3 = 0 \rightarrow x = 3$$

x	$-\infty$	3	$+\infty$
f'	$-$	$-$	
f	↘	↘	

Note that $x = 3$ is not in the domain of f(x) but still must be in the table.
Now:

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$$f'(x) = \frac{-8}{(x-3)^2} = -8(x-3)^{-2} \rightarrow f''(x) = 16(x-3)^{-3} = \frac{16}{(x-3)^3}$$

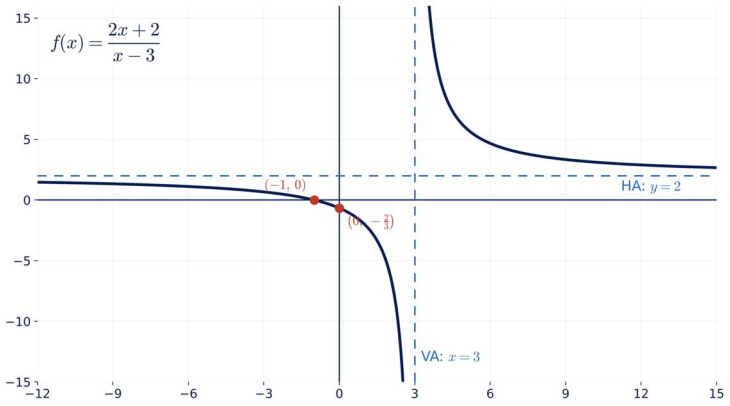
x	$-\infty$ 3 $+\infty$	
f''	−	+
f	∩	∪

Note that $x = 3$ is not an inflection point because it is NOT in the domain of f(x).

For asymptotes:
Vertical Asymptote: Setting denominator equal to zero **à** $x - 3 = 0 \rightarrow x = 3$

Horizontal Asymptote: $\lim_{x\to\infty} \frac{2x+2}{x-3} = \lim_{x\to\infty} \frac{x(2+\frac{2}{x})}{x(1-\frac{3}{x})} = \lim_{x\to\infty} \frac{x(2+0)}{x(1-0)} = 2 \rightarrow \quad y = 2$

Here is the graph:



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4. For finding the inflection point:

$$f(x) = 2x^3 + 6x^2 + 5x - 1 \rightarrow f'(x) = 6x^2 + 12x + 5 \rightarrow$$
$$f''(x) = 12x + 12 = 0 \rightarrow x = -1$$

We know that $x = -1$ will cause a sign change in $12x + 12$ because it is a first order polynomial. And $x = -1$ is also in the domain of f(x) as it is a polynomial. Therefore, it is an inflection point.

$$f(-1) = 2(-1)^3 + 6(-1)^2 + 5(-1) - 1 = -2 + 6 - 5 - 1 = -2$$

$(-1, -2)$ is the inflection point.

Also:

$$f'(-1) = 6(-1)^2 + 12(-1) + 5 = 6 - 12 + 5 = -1 \quad (the\ slope)$$

Now:

$$y - (-2) = -1\big(x - (-1)\big) \rightarrow y + 2 = -1(x + 1) \quad \text{OR} \quad y = -x - 3$$

5. $f(x) = \frac{2.5}{2x^2-1} \rightarrow f'(x) = \frac{0(2x^2-1)-2.5(4x)}{(2x^2-1)^2} = \frac{-10x}{(2x^2-1)^2} \rightarrow$

$$f''(x) = \frac{-10(2x^2-1)^2 - (-10x)(2)(2x^2-1)(4x)}{(2x^2-1)^4} = \frac{-10(2x^2-1) - (-10x)(2)(4x)}{(2x^2-1)^3}$$
$$= \frac{-20x^2 + 10 + 80x^2}{(2x^2-1)^3} = \frac{60x^2 + 10}{(2x^2-1)^3}$$

Now:

$$f''(x) = 0 \rightarrow 60x^2 + 10 = 0 \rightarrow 60x^2 = -10 \rightarrow x^2 = \frac{-1}{6} Not\ Possible$$
$$f''(x) = DNE \rightarrow (2x^2-1)^3 = 0 \rightarrow 2x^2 = 1 \rightarrow x^2 = \frac{1}{2} \rightarrow x = \pm\frac{1}{\sqrt{2}}$$

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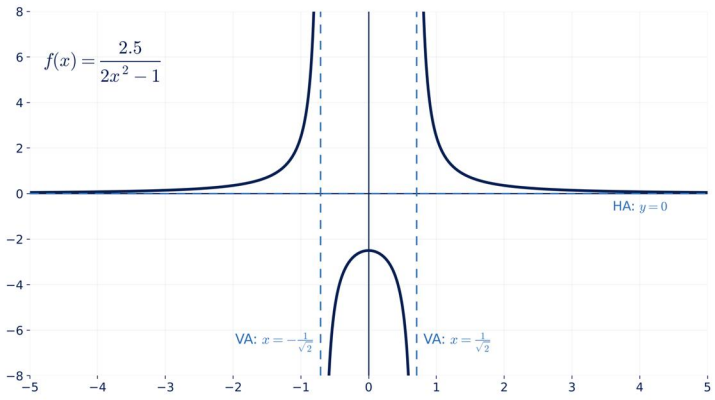
x	$-\infty$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$+\infty$
f''	+	−	+	
f	∪	∩	∪	

Note that $x = \frac{1}{\sqrt{2}}$ or $x = -\frac{1}{\sqrt{2}}$ are not an inflection point because they are NOT in the domain of f(x).

Interval of concave up: $\left(-\infty, -\frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, +\infty\right)$

Interval of concave down: $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

And here is the graph for the reference:



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