Optimization Cheat Sheet

- Steps taken to solve optimization problems:
  1. Form the objective function and the constraint; The constraint might not be very easy to decipher in the very beginning. Even a crude understanding is really helpful.
  2. Start from the constraint and solve for one of the variables. Consider two things here: solve for the one which is computationally easier or the one which is more relevant to the objective function.
  3. Plug that into the objective function such that the objective function is only in terms of one variable.
  4. Take the derivative of it and set it equal to zero to find the optimal value of that variable. Make sure to read the question and see what the end goal is.
  5. Plug the optimal variable that you found in step 4 into the constraint and solve for the other variable.

- Finding the Max or Min
  - Critical points: solve where f′(x) = 0 or f′(x)=DNE
  - Closed interval [a, b]: compare the function values at the critical points and function values at both endpoints
  - If you had more than one critical point use one of the below tests:
    - First derivative test: If f′ changes from positive to negative, it's a max. If it changes from negative to positive, it's a min.
    - Second derivative test [if it exists]: If f″(c) < 0, it's a max (concave down). If f″(c) > 0, it's a min (concave up).
  - Only one critical point on an interval? The local max or min there is the absolute max or min.

- Formulas needed:

Shape	Area	Perimeter
Rectangle	$A = xy$	$P = 2(x + y)$
Circle	$A = \pi r^2$	$P = 2\pi r$

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And also:

Shape	Volume	Surface Area
Square-based Box (open top)	$V = x^2h$	$S = x^2 + 4xh$
Square-based Box (closed top)	$V = x^2h$	$S = 2x^2 + 4xh$
Cylinder (open top)	$V = \pi r^2 h$	$S = \pi r^2 + 2\pi rh$
Cylinder (closed)	$V = \pi r^2 h$	$S = 2\pi r^2 + 2\pi rh$
Cone	$V = \frac{1}{3}\pi r^2 h$	----
Sphere	$V = \frac{4}{3}\pi r^3$	$S = 4\pi r^2$

Ø Distance between two points:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Bonus point: minimize d^2 instead of d

Ø Common Mistakes That Cost Marks:

- ❌ Differentiating before reducing to one variable.
- ❌ Forgetting to check the endpoints on a closed interval.
- ❌ Not justifying max or min if you have more than one x found.
- ❌ Answering with x when the question asks for the area, the dimensions or the cost.
- ❌ Mixing up the constraint and the objective (for example, maximizing the perimeter when the area is fixed).
- ❌ Keeping a critical point outside the domain (a negative length, or a cut bigger than the sheet).
- ❌ Leaving out units in the final answer.

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